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DISSIPATIVE HEATING OF A MEDIUM DURING ROTATION OF A DISK IN A BOUNDED SPACE

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A method is proposed for calculating dissipative heating of a medium in a model of a turbine stage. Results of the calculation are compared with experiment.

Consideration of the problem of flow thermodynamics around rotating axisymmetric bodies grew out of the requirements of power-machine construction to a considerable extent. In the literature up to now the principal attention has been given to problems of hydrodynamic resistance and heat transfer during rotation of bodies in free and bounded spaces [1-3] without analysis of the changes in the parameters of the medium during the irreversible conversion of mechanical energy into heat. At the same time, the enclosure of a rotating body in a bounded chamber presupposes a significant intensification of the effect of dissipation on the thermal state of the medium. There is great interest in the consideration of the rotation of a disk in a cylindrical chamber from this viewpoint, since the flow portion of various turbines contains as a required element alternately arranged rotating and fixed flat surfaces perpendicular to the axis of rotation.

Theoretical and experimental studies of flow hydrodynamics in rotating systems yield only qualitative results in most cases because of the extreme complexity of the processes. Calculations are possible with a number of important simplifications and assumptions which significantly reduce the accuracy of the results.

A rotating disk acts like a centrifugal fan and creates a suction that causes radial motion of the medium from a center near the disk to a center near the chamber walls. In addition to rotation around the axis of the disk and vortex motion in the meridional plane, a certain flow rate of the medium, G_S, ordinarily occurs in the gap between a rotating disk of the turbine and the cylindrical chamber; this is associated with the flow or with the need

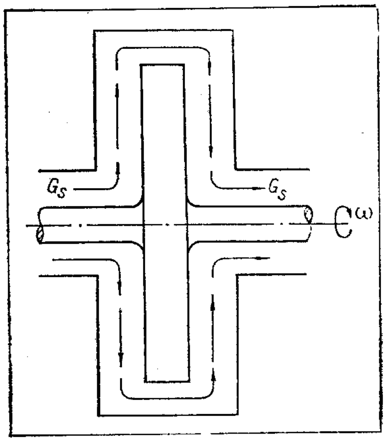

Fig. 1. Diagram of medium flow in gap between surfaces of disk and chamber.

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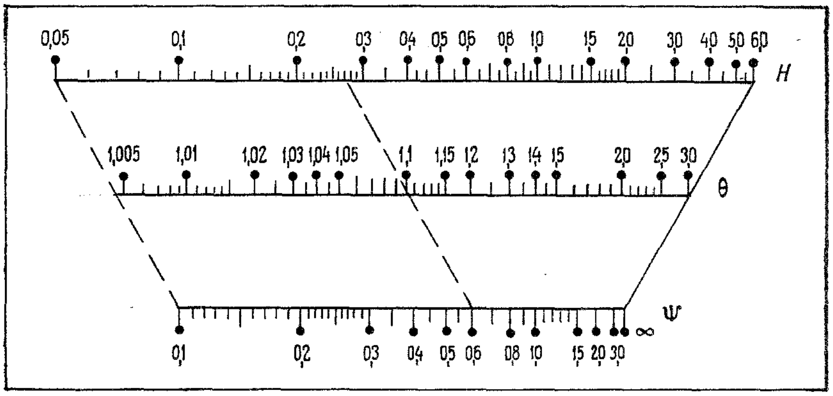

Fig. 2. Nomogram for calculating dissipative heating of a medium during rotation of a disk in a bounded space.

for special cooling. For adequate values of the gap h and for high rates of rotation ω, the formation of separate boundary layers at the disk and at the fixed walls is observed where the main change in the circumferential velocity of the medium occurs. In the core of the flow, a constant value of the circumferential velocity of the medium, $v_\varphi = v_{\varphi*}$, is established at a given radius r.

We estimate the frictional power of a rotating disk for the simplest scheme of axisymmetric flow around the disk with delivery at a given cooling flow rate along the axis on one side of the disk and discharge on the other (Fig. 1).

As in [1], calculation of the increment of angular momentum with respect to the axis of rotation in the circumferential direction for an annular element of the medium between the disk and chamber wall yields the following relations:

$$\frac{d}{dr}\left(2\pi r^2\rho_s\int_0^h v_r v_\varphi dz\right) = 2\pi r^2(\tau_\varphi|_w - \tau_\varphi|_d),$$

$$\tau_\varphi|_d = K_d\frac{\rho_s(r\omega - v_{\varphi*})^2}{2}, \tag{1}$$

$$\tau_\varphi|_w = K_w\frac{\rho_s v_{\varphi*}^2}{2}.$$

One can obtain from Eqs. (1) a relation for the frictional force df acting on elementary annular surfaces of the disk and chamber wall:

$$df_w - df_d = \frac{1}{r}d\left(2\pi r^2\rho_s\int_0^h v_r v_\varphi dz\right),$$

$$df_w = 2\pi r dr\tau_\varphi|_w, \tag{2}$$

$$df_d = 2\pi r dr\tau_\varphi|_d.$$

Neglecting the effect of vortex flow in the meridional plane of the chamber and assuming that the component v_r results only from the flow G_s, a similar relation can be written for the frictional forces on the opposite side of the disk but with the opposite sign on the right side, since the direction of v_r is reversed. The total frictional force for the disk, dF_d, is the sum of these equations. Assuming that the values of K_w and K_d do not depend on the direction of the radial velocity, we obtain

$$dF_d = 2df_d = 2K_d 2\pi r dr\frac{\rho_s(r\omega - v_{\varphi*})^2}{2} = 2K_w 2\pi r dr\frac{\rho_s v_{\varphi*}^2}{2}. \tag{3}$$

Equation (3) indicates that the ratio between the average circumferential velocity of the medium and the circumferential velocity of an annular element of the disk is

$$\varepsilon = \frac{v_{\varphi*}}{r\omega} = \frac{1}{\sqrt{\frac{K_d}{K_w}} + 1}, \tag{4}$$

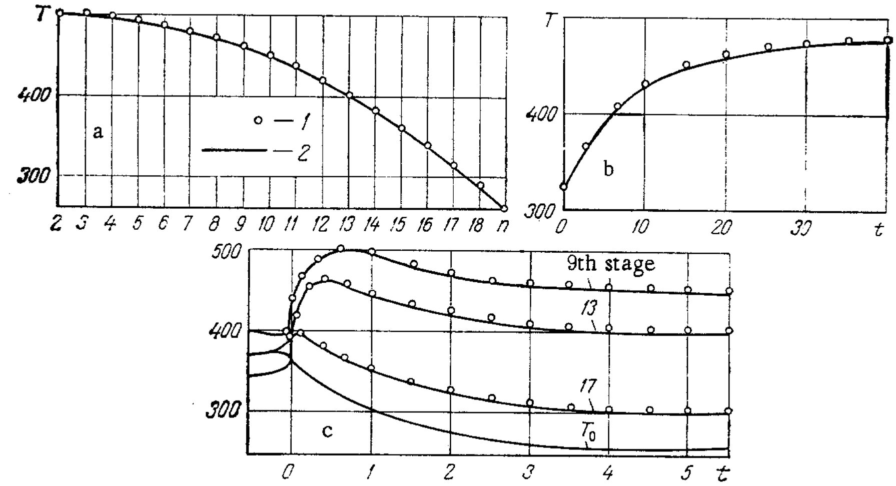

Fig. 3. Comparison of calculated and experimental results [1) calculation; 2) experiment]: a) steady temperature state of flow-through section of P-50-130 turbine in vapor-free mode; n, stage number; b) kinetics of medium heating in individual stage of K-100-90 turbine; t, min; c) kinetics of medium heating along flow-through portion of P-50-130 turbine; t, h; T, °C.

where $\varepsilon = 0.5$ for $K_d = K_w$.

The frictional power is determined from the product of frictional force and velocity [4]:

$$\int_{r_1}^{r_2} dN = \int_{r_1}^{r_2} \omega r dF_d. \tag{5}$$

For known coefficients of friction K_d and K_w assumed constant along the radius, integration of this equation yields

$$N = B\rho_s, \; B = \frac{2\pi K_w \varepsilon^2}{5} \omega^3 (r_2^5 - r_1^5). \tag{6}$$

Because of the number of assumptions, the derived equation is only applicable for rough calculations of the frictional power of rotating systems. Methods given in the literature [3] make it possible in certain cases to solve numerically the equation of angular momentum with allowance for the variability of K_w, K_d, and ε along the radius and also for the effect of the flow rate G_s on them. For complex cases of flow, however, such as in a turbine stage with rotation of a rotor disk in a space of complex configuration with numerous sections of local reverse flows and separation phenomena, it is necessary to turn to empirical equations similar in structure to Eq. (6). A number of empirical formulas obtained by Stodola, Kerr, Forner, etc., which are widely used in power-machine construction, are given in [4].

Because of the work done by frictional force in the medium and at its boundaries, frictional heat is released which heats up both the medium itself and the material in the disk and chamber walls. To simplify the problem, the material in the chamber walls is assumed thermally nonconducting and the thermal resistance of the disk and medium is assumed negligibly small so that their spatial temperature field is uniform. It is further assumed that the pressure forcing the medium through is constant and equal to the pressure in the suction chamber where the heated medium enters. Considering rotation of a disk in a medium which is an ideal gas with allowance for the above statements, one can write the following basic relations for dissipative heating:

$$c_s P_s V_s dT_s + c_d m_d dT_d = N dt + c_s G_s T_{s\,i}\, dt - c_s (G_s + \delta G_s) T_s dt, \tag{7}$$

$$T_s = T_d, \; N = B\rho_s, \; -V_s d\rho_s = \delta G_s dt, \; \rho_s = \frac{P_s}{RT_s}, \; P_s = \text{const}.$$

As boundary conditions, it is assumed the temperature of the medium as the entrance to the chamber is constant and equal to the initial temperature of the disk:

$$T_{\text{д}}|_{t=0} = T_0, \quad T_{s\,\text{en}}|_{t \geq 0} = \text{const} = T_0. \tag{8}$$

As experimental studies have shown [5-7], the limitations assumed correspond to actual relationships in steam turbines for operation in the so-called "vapor-free mode" where the effects of dissipation appear in purest form.

By usual means, one can obtain from Eq. (7) an equation for the temperature of the medium:

$$\left(1+\frac{c_d m_d R}{2c_s P_s V_s}\right)\frac{dT_s}{dt}=\frac{B}{2c_s V_s}+\frac{G_s R T_0}{2P_s V_s}T_s-\frac{G_s R}{2P_s V_s}T_s^2. \tag{9}$$

Equation (9) is directly integrable but the resultant relation $t = t(T_s)$ is unsuitable for analysis and practical calculations because of its inverse functional dependence. It is quite advantageous, then, to convert the original equation (9) to another form and correspondingly obtain its solution in a different form which is more reasonable from the physical aspect. Thus the substitutions

$$1+g_1 T_s=\frac{1}{y},\ y=x(t)\,\eta(\xi),$$
$$x=\exp\int\left(-\frac{U_2}{g_1}\right)dt,\ \xi=\int\left(x\frac{2U_2}{g_1}-U_1\right)dt, \tag{10}$$

where

$$g_1=\frac{c_d m_d R}{2P_s c_s V_s},$$
$$U_1=\frac{G_s R T_0}{2P_s V_s},\ U_2=-\frac{G_s R}{2P_s V_s},$$

bring the solution to the following form according to [8]:

$$T_s=T_0+(T_{s\,\max}-T_0)\left[1-\exp\left(-\frac{c_s G_s}{c_d m_d}t\right)\right],$$
$$T_{s\,\max}=\frac{T_0}{2}+\sqrt{\left(\frac{T_0}{2}\right)^2+\frac{BP_s}{c_s G_s R}}. \tag{11}$$

To facilitate the practical use of the solution (11), a nomogram is given in Fig. 2 for solution in the relative variables θ, H, and Ψ:

$$\theta=\frac{T_s}{T_0},\ \theta_{\max}=\frac{T_{s\,\max}}{T_0},$$
$$H=\frac{BP_s}{c_s G_s R T_0^2},\quad \Psi=\frac{c_s G_s}{c_d m_d}t. \tag{12}$$

The nomogram makes it not only possible to obtain with sufficient accuracy possible temperatures of the medium in the chamber for given parameter values, but also to select these parameters with a view toward producing a desired temperature state of the medium in which the disk rotates.

The acceptability of the assumptions made above and of the resultant equations was checked by a comparison of calculated results and experimental results [5-7] from studies of the temperature state of the steam in the flow-through stages of P-50-130 and K-100-90 turbines from the Leningrad Metal Plant during their operation in the vapor-free mode at a given cooling flow rate. The frictional power of the rotor disk in the stage was computed from the empirical formula of Stodola [4]. As is clear from Fig. 3, the agreement of the results is completely acceptable for engineering calculations.

NOTATION

T, θ, temperature; P, pressure; V, volume; m, mass; G, flow rate; ρ, density; c, specific heat; v, linear velocity; ω, circumferential velocity; K, coefficient of friction; τ, tangential stresses; f, F, forces; R, gas constant; N, power; t, time; r, φ, z, coordinates. Indices: d, disk; w, wall; s, medium; i, inlet; 0, initial; max, maximum.

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EFFICIENCY OF CONVECTIVE CIRCULAR FINS WITH A TRIANGULAR PROFILE

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We propose a graph for determining the efficiency factor of circular fins with a triangular profile, as well as a table of the principal parameters of such fins with minimum mass.

For circular fins with a triangular profile, of the kind shown in Fig. 1, the convective heat flux through the base is given by the formula

$$Q_0 = 2\pi r_1 l \alpha \vartheta_0 (\varphi + 1)\eta = -2\pi r_1 \frac{\delta_0}{l} \lambda \vartheta_0 \theta_0' , \tag{1}$$

where $\varphi = r_2/r_1$, θ_0'' is the dimensionless temperature gradient near the base of the fin, and η is the efficiency factor of the fin, which depends on the geometric shape of the fin and on the dimensionless parameter

$$\sigma = \frac{2\alpha l^2}{\lambda \delta_0} = 2\,\mathrm{Bi}\left(\frac{l}{\delta_0}\right)^2 . \tag{2}$$

The function $\eta(\sigma)$ is defined by the relations in Eq. (1):

$$\eta = -\frac{2\theta_0'}{(\varphi + 1)\sigma} . \tag{3}$$

In order to determine η we need a solution of the differential equation of the temperature field, which ($\rho = r/l$; $\Lambda = \delta/\delta_0$; $\theta = \vartheta/\vartheta_0$), on the assumption that the heat flux is propagated only in the radial direction (one-dimensional problem), has the form

$$\Delta \frac{d}{d\rho}\left(\rho \frac{d\theta}{d\rho}\right) + \frac{d\Delta}{d\rho}\left(\rho \frac{d\theta}{d\rho}\right) - \sigma(\rho\theta) = 0. \tag{4}$$

For triangular and trapezoidal profiles the dimensionless thickness Δ of the fin can be expressed linearly in terms of ρ:

$$\Delta = \frac{\varphi}{\varphi - 1} - \rho. \tag{5}$$

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